

- I. *An Account by John Eames, F. R. S. of a Dissertation, containing Remarks upon the Observations made in France, in order to ascertain the Figure of the Earth, by Mr. Celsius, intituled, De observationibus pro Figura Telluris determinanda, in Gallia habitis, Disquisitio. Auctore Andrea Celso, in Acad. Upsal. Astronom. Prof. Regio, &c. Upsalæ, 1738. 4^{to}.*

THAT the Figure of the Earth is Spheroidal is agreed upon by all: But whether it be an oblong or oblate Spheroid, *i. e.* whether the Axis be longer or shorter than a Diameter at the Equator, has been for some time a matter of Doubt. Three several Methods have been proposed to determine this Controversy by Experiments; as by the different Lengths of Pendulums vibrating Seconds, in different Latitudes; the Figure of the Earth's Shadow in Lunar Eclipses; and by the actual Measurement of the Lengths of a Degree on the Meridian in different Latitudes.

It is certain, if the Lengths of the Degrees of Latitude decrease as we go from the Equator toward the Poles, then the Axis is greater, and the Figure an oblong Spheroid; but, on the contrary, if these Lengths increase as you remove towards the Poles, the Axis is less than a Diameter at the Equator, and consequently an oblate Spheroid.

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Mr. *Cassini* and others, judge the Earth to be of an oblong Spheroidal Figure; and the Observations made in *France*, if intirely to be depended upon, prove this *Hypothesis* to be a Matter of Fact. Our late illustrious President, Sir ISAAC NEWTON, Mr. *Huygens*, and others, make the Earth to be an oblate Spheroid, higher at the Equator than at the Poles; and this Figure of the Earth is undoubtedly the true one, if the Observations lately made near the Arctic Circle be admitted as certain and exact. So that since both Sets of Observations have been taken by Persons of known Skill, Dexterity, and Integrity, it is now become absolutely necessary to inquire into this Matter, in order to find out the Occasion of so great a Difference in their Conclusions.

Mr. *Celsius*, in the Treatise before us, proposes to consider this Matter more closely, and begins with a Defence of the Observations made at *Tornea*, near the North Polar Circle; and then takes Notice of some things, proper to be considered, relating to the Instruments, Astronomical Observations, and Trigonometrical Operations, performed in *France*; which, in his Judgment, render the Observations uncertain; at least so far as not to be accurate enough to be depended upon in determining the Matter in Question.

To begin with the Defence of the Observations made at *Tornea*: Perhaps it may not be improper to premise a short Account of them. They were undertaken at the Charge of the King of *France*, by Five skilful Gentlemen; Three of them Members of the *Royal Academy* at *Paris*, who were joined by Mr. *Celsius*, and the Abbé *Authier*. The Trigonometrical
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Part of the Work was performed near the River of *Tornea*, whose Direction is the same with the Meridian of *Tornea*; the Coasts of the Gulph of *Bothnia* being found very inconvenient for that Purpose. By the favourable Situation of Five Mountains they formed Eight Triangles, which took in Space enough for their Design. All the Five Gentlemen observed, one after another, each Angle of these Triangles, setting them down in writing separately.

They afterwards determined the Distance between *Tornea* and Mount *Kittis*, under the same Meridian, by a Basis, measured on the River when frozen over, whose Length was 7406 Toises 5 Feet, by the first Measurement; and when measured again, was barely Four Inches over. This Distance between them they found to be 55,234 Toises.

The first Part of their Work being thus finished, the next was to find the Difference of Latitude of these two Places: This they did by the Help of a Telescope, fixed to a Sector of Nine Foot, made at *London*, by the Care and Direction of Mr. *George Graham*, to whom the Lovers of Astronomy are indebted for the curious and well-contrived Instruments he has supplied them withal. The Star they observed at *Tornea* was *α Draconis*: They repeated their Observations three times, and the greatest Difference between them was but Two Seconds: Removing to Mount *Kittis*, they took the same Number of Observations, of the same Star, without finding more than One Second Difference. The Result was, that the Amplitude of the Arch, in the Heavens, between *Tornea* and Mount *Kittis*, (allowing for the Precession of the Equinox, and the Time elapsed between

the Two Observations, according to Mr. *Bradley's* Theory) was 57° Minutes 26 Seconds. Hence the Magnitude of a Degree, on the Earth, intersecting the Polar Circle, was found to be greater than a mean Degree of *France* 377 Toises; and to differ 900 Toises from what it should have been, according to Mr. *Cassini's* Hypothesis: And if the Correction, according to Mr. *Bradley's* Theory, were omitted, the Difference would have amounted to above a Thousand Toises: The Consequence of which, say the curious Observers, is, That the Earth is not only flatted towards the Poles, but that it is much more so than Sir *Isaac Newton* or Monsieur *Huygens* thought it. This unexpected Difference being so very great, made them resolve upon a careful as well as new kind of Verification of the Whole. In the first Place, they repeated their Astronomical Observations Three several times, at *Tornea* and *Kittis*, with the same Instrument, but on another Star, *viz.* δ *Draconis*: The Difference of Latitude between the Two Places was found to be the same, within Three Seconds and an half, with the First. They then not only examined the Truth of their Meridian Line, the Exactness of the Sector, in the different Divisions upon the Limb, chiefly in the Two Degrees imployed in observing α & δ *Draconis*, but supposed that, in their Trigonometrical Operations, they had erred in each Triangle, by Twenty Seconds in each of the Two Angles, and Forty Seconds in the Third; and that all these Errors tended to diminish the Length of the Arch; the Calculation, upon this Supposition, gives but $44 \frac{1}{10}$ Toises for the greatest Error that could be committed.

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When a particular Relation of all these Observations was read before the *Royal Academy of Sciences* at *Paris*, and inquired into; the main Exception taken to them was, That the Observers, omitting to make a Proof of the Line of Collimation, by means of double Observations, with the Face of their Instrument turned contrary Ways, have thereby not duly ascertained the Truth of their Observations. But this Objection was fully answered by Monsieur *Maupertuis*, as Mr. *Celsius* hopes and believes, to the intire Satisfaction of Monsieur *Cassini*, who made it. He allows M. *Cassini* had very good Reason to mention this, as a thing proper to be done in Instruments of common Use, for this Purpose, which generally stand in need of such a Method of Verification: But it was not at all necessary in the Instrument used at *Tornea* and Mount *Kittis*: The very Make of it was such, that no Alteration could easily be made in it, so as to create any perceptible Error in the Observations. The whole Apparatus of the Telescope and Sector is all framed together; the Object-glass and Cross-wires, as well as the Limb, so firmly fixed to the Tube, as not to be dislocated without great Violence. Notwithstanding all this, the utmost Care was taken in transporting it from one Place to another; being placed in a Chest, that the *Lapllanders*, to use his own Words, *in illa cista idolum quoddam servari facile sibi persuaderent*. He adds, the same Objection may be made to Monsieur *Picard's* Observations, who does not seem to have used this Precaution, as Monsieur *Cassini* himself acknowledges, who nevertheless approves and extols his Observations for their Accuracy: So that those at the Arctic Circle

Circle may be very good, notwithstanding the want of this, supposed necessary, Operation. And indeed, that they were so, sufficiently appears from this Fact. The Difference of Latitude between *Tornea* and Mount *Kittis*, found in *September*, was observed again in *March* following, by the Help of the same Star δ *Draconis*, and did not differ from the former above $3\frac{1}{2}$ Seconds, though the Instrument had been twice carried from one Place to the other. This is a Degree of Exactness not easy to be met with; no not in Monsieur *Cassini's* Observations, made on different Stars, which differ sometimes 40 Seconds, in determining the Amplitude of an Arc in the Heavens, though their Instrument was carefully examined in the way above-mentioned.

The Author then proceeds, in his Turn, to inquire into the Accuracy and Certainty of the two Sets of Observations made in the North and South Parts of *France*, in respect of the Royal Observatory at *Paris*.

As to the Measures of the Degrees in the Northern Parts of *France*, between *Paris* and *Dunkirk*, he owns they cannot be much out of the way; being in some measure confirmed by Monsieur *De la Hire*, in the Year 1683. and Monsieur *Cassini* himself. Yet Mr. *Celsius* observes, that the Basis on the sandy plain Shore, near *Dunkirk*, when measured again, differed Three Feet from the former Measurement; which is a much greater Difference than that Mr. *Celsius* and the other Gentlemen found, in measuring a much longer Line twice over, which was but Four Inches.

As to the Astronomical Observations taken by the Six Foot Sector, whose Limb of 12 Degrees was divided only at every 20 Seconds; it is true, Monsieur
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Cassini examined the Instrument several ways, at *Paris*, after his Return thither: but that a Correction, owing to the Change of Centre, might be safely applied to the Observations at *Dunkirk*, the Examen of the Centre should also have been taken at *Dunkirk*; it being uncertain, whether this Alteration or Aberration of the Centre was caused by the Journey to or from *Dunkirk*.

The Difference of 41 Seconds between the Observations taken to settle the true Measure of the Arc of the Heavens, seems to be enormous. Perhaps the Stars were not lucid enough to be well observed by the Three Foot Tube; but might they not, for a due Degree of Accuracy, have been viewed through the Nine or Ten Foot Telescope?

Our Author prefers the Observations of 1719, made after the Return to *Paris*, to those made before; because made at the same time of the Year with those of *Dunkirk*, and so not standing in need of Mr. *Bradley's* Correction: Though this Caution, perhaps, may be thought not necessary here, where the Errors of the Observations are greater than the Correction itself. Mr. *Celsius* remarks farther, if the Difference of Latitude between *Dunkirk* and *Paris* be supposed to be Two Degrees 12 Minutes 12 Seconds and an half, which is a Mean between Four others he mentions, the Length of a Degree will amount to but 56,395 Toises. And if the Observations at *Malvoisine* and *Amiens*, be counted according to Mr. *Bradley's* Theory, for the Interval of a Month between the Observations, the Length of a Degree will come out to be 56,926 Toises; which is 135 Toises less than the Length of a Degree, found
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by measuring the whole Length of *France*; and 134 less than that of Mr. *Picard*, so highly approved of by Mr. *Cassini*, as confirming his own.

The Reflections Mr. Celsius makes upon the Observations, &c. taken in the South Parts of France, shall be referred to another Meeting.

The Continuation of the Remarks made by Mr. Celsius on the Observations taken in France, in order to ascertain the true Figure of the Earth.

MR. *Celsius* having finished his Remarks upon the Observations made in the North Part of *France*, extending from *Paris* to *Dunkirk*, proceeds to examine those taken in the South, from *Paris* to *Collioure*, near the Borders of *Spain*, and the *Pyrenean* Mountains. By the former, a mean Degree was found to consist of 56,960 Toises, by the latter 57,097; and consequently the Earth is an oblong Spheroid.

Mr. *Celsius*, in examining these Observations, which were taken under the Conduct and Direction of the late M. *Cassini* in 1700, first considers the Structure and Goodness of the Instruments used; then the Accuracy of the Astronomical Observations for finding the Difference of Latitude; and, in the last place, the Trigonometrical Operations for determining the Distances of Places; especially the two Extremes under the same Meridian,

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The principal Instrument M. *Cassini* carried with him, was, a Limb of 12 Degrees, whose Radius was indeed 10 Foot, but divided only into Degrees and Minutes; the other Parts were added to it at *Perpignan*. Here Mr. *Celsius* observes, that the finding the true Centre of this Limb was and still is a very difficult and troublesome Problem to a good Artist; that no mention is made, whether the Position or Place of this Centre, and the Divisions of the Limb, were ever examined at *Paris* or *Collioure*, though the Carriage of the Instrument through so long and rough a Way, could not but make some Alteration in the Place of the Centre.

It is true, the Zenith Distance of *Capella*, taken by it at *Paris*, was confirmed to be right by another Instrument; but it cannot be concluded, that the Zenith Distance of the same Star, taken at *Collioure* by this Instrument, and not confirmed there by another Instrument, must be true also. For the Point of Division, answering to this Distance in the Limb, was not examined; and a Centre wrong placed may by Accident give the true Zenith Distance, *viz.* when the true and erroneous Centre happen to lie in the same Perpendicular to the Horizon.

The Exceptions taken to the Astronomical Observations for finding the Difference of Latitude between *Paris* and *Collioure*, are, in the first place, That though Five Stars were observed at *Collioure* and *Paris*, yet One only was made use of, *viz.* *Capella*: That the Difference of Latitude by *Capella* is $6^{\circ} 18' 57''$: If *Lucida Lyra* had been used, the Difference would have been but $6^{\circ} 17' 7''$; but by the Right Shoulder of *Auriga*, $6^{\circ} 19' 25''$: Hence arises the

Uncertainty or Difference of $2' 18''$ between the greatest and least of their Observations: That the late Mr. *Cassini* makes the Difference $57''$ less than Mr. *Cassini*, who accounts for this Difference from the Observations being taken by an ordinary Instrument; but the Instrument is the same which was used to take the Altitude of the Pole of *Amiens*, which was very near that found by Mr. *Picard*.

As to the Trigonometrical Operations for finding the Distance of Places, Mr. *Celsius* thinks they labour under considerable Uncertainties; not only on the Account of the many Difficulties they met withal, *viz.* mountainous Countries, want of proper Signals, &c. so that convenient Triangles could not be formed; but add to all these, several of the Triangles had but Two Angles observed, and some of these Angles too acute; whence, as Mr. *Cassini* himself very justly observes, in his Examination of *Snellius* and *Riccioli's* Observations, great Errors may arise. Mr. *Picard* thinks all Angles less than 20 Degrees ought to be avoided; as also that the Triangles should be contrived so as to have Sides of a due Length, neither too great nor too small: Then follow 16 Triangles, wherein one or more of these Inconveniences are to be found.

It may be said, the Whole of these Observations and Measures of Monsieur *Cassini* seem to be sufficiently confirmed, if not ascertained; since the principal Base in *Roussillon* was found, when computed, to differ but Three Toises from the same as it was actually measured; and that, after some due Corrections, it was made to agree with the greatest Exactness. Mr. *Celsius* replies, Why are we not told
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what those Corrections were, that we may see whether they were really necessary or no? Why were they not taken notice of in the Calculations of each Triangle? Besides, the real Length of the Base, or the fundamental Line, in *Roussillon*, is not fully ascertained, it not being measured more than once; whereas that at *Dunkirk* and that of Mr. *Picard* were measured twice; and there was more Reason for doing so here than at *Dunkirk*, on account of the uneven and almost ever changing Shore in *Roussillon*, from the restless overflowing Sea.

The great Number of the Triangles, joined with the numerous small Errors of the Angles, is another Ground of Uncertainty; for the Errors in the Angles, though small, may make the Distance of the Parallels of the Two extreme Places greater than it ought to be; and yet the principal Sides, that is, those that are made Bases to the following Triangles, continue the same. This made it necessary to verify the Sides, at least at every second Degree, by measuring the principal Base twice over with due Care; which might have been done, and therefore should have been done, in a Matter of so much Nicety as an Attempt to find the Difference between Two Degrees so near one another, under the same Meridian.

To shew what bad Consequences may arise from small Errors committed in observing the Angles of several Triangles, Mr. *Olavus Hiorter*, a curious and ingenious Friend of Mr. *Celsius*, has taken the Pains to form the Triangles of Mr. *Cassini* between *Bourges* and *Collioure*; so that the Distance between their Parallels shall be considerably lessened; and yet the Base in *Roussillon*, found by Computation, shall not, after due Correction, differ sensibly, if at all, from

the same actually measured. In consequence of this, Mr. *Celsius* concludes with observing, that the Distance between the Royal Observatory and the Perpendicular to the Meridian of *Collioure*, deduced from the Triangles of *Cassini*, corrected after Mr. *Hiorter's* Method, &c. will amount to but 358,980 Toises. This, divided by the mean Difference of their Latitudes, $6^{\circ} 19' 11''$, will give 56,803 Toises, for the Length of a Degree, one with another, between *Paris* and *Collioure*, which is less than the Length of a mean Degree found by Mr. *Picard*, and pretty near the Truth: So that the Degrees decrease as you go towards the Equator; and consequently the Earth is higher at the Equator than at the Poles, as Sir *Isaac Newton* and Mr. *Huygens* believed.

The Distance of the Parallels of *Paris* and *Collioure* by this Method is indeed less than that computed by Mr. *Cassini*; but this cannot reasonably be complained of, since these computed Measures of Mr. *Cassini* seem very capable of being lessened; and it is no more than what Mr. *Cassini* himself hath done to the Measures published by his Father, which he has shortened by $325\frac{1}{2}$ Toises. But however that Matter be, whether this particular Correction of Mr. *Cassini's* Distance, and, consequently, Length of a mean Degree, be admitted or no, Mr. *Celsius* is fully persuaded, upon the Whole, that he hath made it plain to every unprejudiced Reader, that these Two Sets of Observations in *France* are not taken with such a Degree of Exactness as to be depended upon, in determining so nice a Matter, in Dispute for 50 Years, as the true Figure of the Earth; which was the thing proposed to be done by them.